

# Estimation of the Random Error Variance of Measurements on Time-Varying Values

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- Weighted Least Square (WLS) State Estimation

$$\min \sum_{k=0}^n w_j \left( z_j - z_j(x) \right)^2$$

- Principle for selection of  $w_j$

$$w_j = \frac{1}{\sigma_j^2}$$

- Meaning of  $w_j$ :
  - higher accuracy, greater  $w_j$



- Setting of measurement weigh by type, e.g. (per unit value)
  - Weigh of P: 5.0
  - Weigh of Q: 2.0
  - Weigh of V: 10.0
  - ...
- Seems reasonable, actually arbitrary!
- Still applied in some online SE software!



- Calculation of the weigh through available information on the accuracy
  - Measurement full scale ( $f$ )
  - Measurement accuracy class ( $p$ )

- Formulation

$$\sigma = \frac{fp}{3 * 100}$$

- Reference

- DOI: 10.7500/AEPS20150615015



- Illustration of per unit weigh calculated from full scales and accuracy levels

kV	Measurement Weigh (per unit)		
	P	Q	V
110	7695	1924	6944
500	85	21	6944



- Real situation
  - For P&Q measurements: great difference at different voltage levels
  - No direct correlations between the P/Q & V measurement weigh
- Weigh setting by measurement type is far from reality



- To get more reliable accuracy information on each measurement: error variance

$$s^2 = \frac{1}{n-1} \sum_{i=0}^n (z_i - \bar{z})^2$$

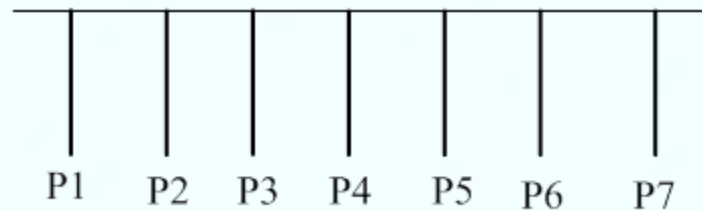
- $z_i$ s are should be taken from a constant value.
- In power system, the variance can be calculated only through offline way.
- **Can the variance be estimated online and from time-variant values ?!**





- To construct constant value
- Typical case: zero injections in to a logical bus

$$c = \sum_{i=0}^7 P_i \equiv 0$$



- Relation between variance of  $c$  and variance of  $P_i$

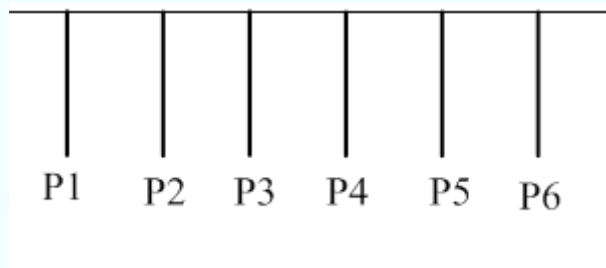
$$S_c^2 = \sum_{k=1}^7 S_{P_i}^2$$

- Useful deduction
  - $S_{P_i}^2 < S_c^2$
  - $S_{P_i}^2 = S_c^2/7$ , with the assumption that all  $S_{P_i}^2$  are equal



- Furthermore, if p7 is switched off from the bus at some time

$$c' = \sum_{i=0}^6 P_i \equiv 0$$



- Update of the variance relation

$$S_{C'}^2 = \sum_{k=1}^6 S_{P_i}^2$$

- **New and very valuable deduction**

$$S_{P_7}^2 = S_C^2 - S_{C'}^2$$

- By this way, all  $S_{P_i}^2$  can be estimated



- If we can construct a linear transformation from  $n$  time-variable  $v_i$  to  $m$  time-invariant values  $C_j$

$$C = Av$$

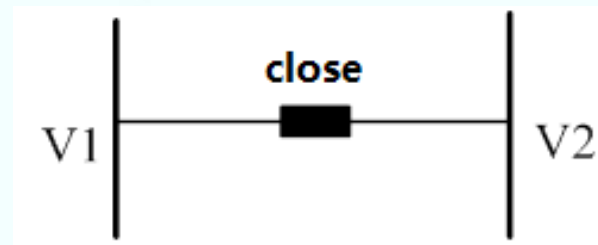
- and  $S_C^2$  and  $S_v^2$  are the variance of  $C$  and  $v$ , it follows

$$S_C^2 = BS_v^2, \quad B_{ij} = A_{ij}^2 \geq 0$$



- If  $\text{rank}(B) = m = n$ 
  - $S_v^2 = B^{-1}S_C^2$
  - As  $C$  is time-invariant,  $S_C^2$  can be estimated through multiple measurements.
  - **$S_v^2$  then can be estimated indirectly.**
- If  $m > \text{rank}(B) = n$ 
  - Least Squared  $S_v^2$  can be estimated,  
$$S_v^2 = (B^T B)^{-1} B^T S_C^2$$
- If  $\text{rank}(B) < n$ 
  - $S_v^2$  can be estimated with some other assumption, e.g. minimum difference among the element of  $S_v^2$ .

- For zero power injection, **system error** of the each element measurement can be estimated through similar way.
- Certain linear combinations can be constructed for bus bars that are connected together, so that the measurement error variance can be estimated for **voltage magnitude and phase**.



- Well verified by site data.
- More data will be collected to evaluate both the method and measurement quality.
- Estimation of measurement error variance is separated from the main SE problem.
- The results of variance estimation will provide more solid base for online SE.





# Thanks

# Q&A

